Reparametrization invariance reexamined

HUTP-96/A053
hep-ph/9701243
January 24, 1997

Reparameterization Invariance Revisited

Markus Finkemeier, Howard Georgi, Matt McIrvin

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138, USA

Abstract

Reparameterization invariance, a symmetry of heavy quark effective theory, appears in different forms in the literature. The most commonly cited forms of the reparameterization transformation are shown to induce the same constraints on operators that do not vanish under the equation of motion to order $1/m^2$, and to be related by a redefinition of the heavy quark field. We give a new, very straightforward proof that that the reparameterization invariance constraints apply to all orders in α_s under matching to full QCD and renormalization-group running, at least up to and including $O(1/m^2)$.

1 Introduction

Heavy particle effective theories are useful in a variety of situations [1, 2, 3, 4, 5, 6, 7, 8]. In these effective theories, S matrix elements are expanded around the limit $1/m \to 0$, in which limit the heavy particle becomes nearly static and the velocity v of the heavy particle becomes a conserved quantum number. The momentum p of the heavy particle is decomposed as

$$p = mv + k \tag{1}$$

where m is the mass of the heavy particle, and v is a four velocity ($v^2 = 1$), which must be choosen such that the residual momentum k is small compared to m. Clearly, the decomposition $p = mv + k$ is not unique (see [9], for example). We can as well write $p = mv' + k'$ where $k' = k + (v - v')/m$, as long as $v'^2 = 1$.

This leads to the requirement of reparameterization invariance for the effective Lagrangian [10, 11, 12]. In the case of a scalar field $\phi(x)$ [8, 10], the issue is rather simple. Let us consider an infinitesimal reparameterization

$$v \to v' = v + \delta v \qquad \text{where} \qquad v \cdot \delta v = 0 \tag{2}$$

The effective Lagrangian $\mathcal{L}_v$ is written in terms of $\phi_v(x)$, defined by

$$\phi_v(x) = \sqrt{2m}\exp(imv \cdot x)\phi(x) \tag{3}$$

and the reparameterization (2) leads to

$$\phi_v \to \phi'_v = \exp(im\delta v \cdot x)\phi_v = [1 + im\delta v \cdot x]\phi_v \tag{4}$$

Due to the appearance of m in the transformation law (4), the requirement of invariance of the effective Lagrangian under reparameterization leads to relations between couplings of different order in $1/m$.

In the case of spin 1/2, the situation is more complicated, because the reparameterization transformation of the field $\Psi_{+v}(x)$ must involve a rotation in Dirac space, in order to ensure that the projection identity $\not{v}\Psi_{+v}(x) = \Psi_{+v}(x)$ is transformed into $\not{v}'\Psi_{+v}(x) = \Psi_{+v'}(x)$.

Indeed, there is controversy in the literature on the correct form of the reparameterization transformation for heavy quark effective theory. In their paper on the issue, Luke and Manohar [10] propose a certain form for this transformation $\Psi_{+v} \to \Psi_{+v'}$. This transformation law has been criticized by Yu-Qi Chen [11] as being incorrect at $O(1/m^2)$.

Chen proposed a different transformation law, and in fact it is straightforward to calculate that the effective Lagrangian obtained from tree level matching to full QCD is invariant under Chen's transformation, but not under the one proposed by Luke and Manohar. This alone, however, does not imply that Luke and Manohar's transformation law is incorrect, because the form of the effective Lagrangian is not unique. Field redefinitions of the heavy quark field can change the Lagrangian without changing the physical predictions.

The purpose of this paper is to shed some light on this question. In fact, we have not been able to follow the arguments in either [10] nor in [11] regarding the derivation of the reparameterization transformation step by step. To which extent this is due to our own inabilities, and to which extent the arguments are actually inconclusive or wrong, is not completely clear to us at each point, either. Therefore we decided to investigate the issue on our own along somewhat different lines.

Our main results are as follows. The difference between Chen's transformation and Luke and Manohar's, at least to order $1/m^2$, has to do with the presence of the "class II operators" that vanish under the leading-order equation of motion. They are therefore members of a family of reparameterization transformations which, interpreted as symmetries, impose the same constraints on the coefficients of the "class I operators" that do not vanish under the leading-order equation of motion. We have found a new, very straightforward proof that these constraints in fact hold to order $1/m^2$ in the heavy mass expansion, not merely at level, but to arbitrary order α_s^n in QCD perturbation theory. We also prove that the constraint imposed by Chen's transformation on class II operators holds to order $1/m^2$ but all orders in α_s, if one uses the field definitions obtained via our straightforward type of matching.

One might suspect that analogous relations will hold at higher order $1/m^3$, but we have not proven that. Furthermore, it is not really clear to us that reparameterization invariance constraints will hold unchanged for non-perturbative effects. We would like to encourage further study of this issue.

The present paper is organized as follows.

In Sec. 2, we review the structure of the heavy quark effective theory Lagrangian. In Sec. 3, we show how Chen's transformation law can easily be derived on a classical level. We find that the tree level matching Lagrangian is invariant under this transformation law. Then, in Sec. 4, we compare this with Luke and Manohar's transformation. We show that the two transformation laws differ by a redefinition of the fields.

In Sec. 5, we discuss reparameterization invariance constraints on the couplings of the effective Lagrangian, and a subtlety in applying the statements in [10] to the relations between the coupling coefficients at order $1/m^2$. There has been some confusion over the implications of Luke and Manohar's version of reparameterization invariance. We show that Luke and Manohar's transformation actually yields the same class I constraints as Chen's.

In Sec. 6, we discuss loops and matching corrections. The Wilson coefficents $C_i(\mu)$ which multiply the various operators in the effective Lagrangian can be obtained in a two-step process. In the first step, "matching", one can use a renormalization scale $\mu = m$. The $C_i(m)$ are then determined by requiring Green's functions in the full and the effective theory to be equal. In the second step, "running", the renormalization group equations are used to evolve down from m to scales $\mu \ll m$. We discuss why the invariance under Chen's transformation is preserved to all orders in α_s and up to (including) order $1/m^2$ in these two steps. In Sec. 7 we draw our conclusions.

2 Operators in the heavy quark Lagrangian

The general form of the heavy quark effective Lagrangian is given by [13, 14]

$$\mathcal{L}^{\text{eff}} = \overline{\Psi}_{+v} iD \cdot v \Psi_{+v} + C_{kin} O_{kin} + C_{mag} O_{mag} + C_1 O_1 + C_2 O_2$$

$$+\,(\text{class II terms}) + O(1/m^3) \tag{5}$$

where

$$\begin{aligned}
O_{kin} &= -\frac{1}{2m}\overline{\Psi}_{+v}D^2\Psi_{+v} \\
O_{mag} &= \frac{g}{4m}\overline{\Psi}_{+v}\sigma^{\mu\nu}G_{\mu\nu}\Psi_{+v} \\
O_1 &= \frac{g}{8m^2}\overline{\Psi}_{+v}v^{\mu}[D^{\nu}, G_{\mu\nu}]\Psi_{+v} \\
O_2 &= \frac{ig}{8m^2}\overline{\Psi}_{+v}\sigma^{\alpha\mu}v^{\nu}\{D_{\alpha}, G_{\mu\nu}\}\Psi_{+v}
\end{aligned} \tag{6}$$

We define $D_\mu = \partial_\mu - igA^a_\mu T^a$, and $G_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$.

We have chosen to define the operators O_i such that tree level matching to full QCD yields $C_1 = C_2 = C_{kin} = C_{mag} = 1$. There is more freedom in defining the class I operator basis, which comes from the ability to add or remove class II terms from these operators. The definitions above make quantum corrections and field redefinitions easy to deal with.

Class II operators have the general form

$$O_i = \overline{\Psi}_{+v}(iD\cdot vA + \overline{A}iD\cdot v)\Psi_{+v} \tag{7}$$

and so vanish when applying the classical equations of motion. They can be removed from the effective Lagrangian by a field redefinition which does not change the coefficients of the class I operators C_{kin}, C_{mag}, C_1 and C_2. A convenient basis of operators to work with is

$$\begin{aligned}
O_{D\cdot v} &= -\frac{1}{2m}\overline{\Psi}_{+v}(iD\cdot v)^2\Psi_{+v} \\
O_a &= \frac{1}{4m^2}\overline{\Psi}_{+v}\{iD\cdot v, (iD)^2\}\Psi_{+v} \\
O_b &= \frac{1}{4m^2}\overline{\Psi}_{+v}(iD\cdot v)^3\Psi_{+v} \\
O_c &= \frac{-g}{8m^2}\overline{\Psi}_{+v}\{iD\cdot v, G^{\mu\nu}\sigma_{\mu\nu}\}\Psi_{+v}
\end{aligned} \tag{8}$$

Upon tree-level matching, $C_{D\cdot v} = -1$,$C_a = -1/2$, $C_b = 1$, and $C_c = 1$. (This basis was chosen to make it easy to translate results about renormalization-group running from [15].)

We have used a basis of Hermitian operators. Its most important feature is that the class I and class II parts of the Lagrangian are separately Hermitian. This makes it possible to remove class II operators with field redefinitions, as in [14] and below.

It is purely for convenience that we define the *individual* operators to be Hermitian, but if one uses a non-Hermitian operator basis and wishes to leave in the class II part, it is important to include enough operators to be able to reproduce all of these Hermitian operators by taking linear combinations. (See our comments on [16] in Sec. 5.1 below.)

3 Derivation of Chen's transformation law at the classical level

Let's start from the heavy quark effective theory using v. One defines

$$\Psi_{\pm v} = e^{imv\cdot x}\frac{1 \pm \not{v}}{2}\Psi(x) \tag{9}$$

where $\Psi(x)$ is the quark field that appears in the QCD Lagrangian. This implies

$$\Psi(x) = e^{-imv\cdot x}[\Psi_{+v}(x) + \Psi_{-v}(x)] \tag{10}$$

The tree level matching Lagrangian is obtained by integration out the heavy field Ψ_{-v} using the classical equations of motion

$$\Psi_{-v} = \frac{1}{2m + iv\cdot D}i(\not{D} - v\cdot D)\Psi_{+v} \tag{11}$$

Now consider a effective theory using v' with

$$v \to v' = v + \delta v \qquad \text{where} \qquad v\cdot\delta v = 0 \tag{12}$$

We can express $\Psi_{+v'}$ through Ψ_{+v} by using the classical equations of motion. We have

$$\begin{aligned}
\Psi_{+v'} &= e^{imv'\cdot x}P_{+v'}\Psi \\
&= e^{im(v+\delta v)\cdot x}\frac{1 + \not{v} + \delta\not{v}}{2}e^{-imv\cdot x}\Big[1 + \frac{1}{2m + iv\cdot D}i(\not{D} - v\cdot D)\Big]\Psi_{+v} \\
&= \left[1 + im\delta v\cdot x + \frac{\delta\not{v}}{2} + \frac{\delta\not{v}}{2}\frac{1}{2m + iv\cdot D}i(\not{D} - v\cdot D)\right]\Psi_{+v}
\end{aligned} \tag{13}$$

which is Chen's transformation law [11] (see also [12]).

In the above derivation of the transformation law, we have used the classical equations of motion for Ψ_{-v}. It is therefore not obvious whether matching corrections to the effective Lagrangian will be invariant under the transformation law.

Furthermore, something funny has happened. In the effective theory based on v, the two "heavy components" Ψ_{-v} are integrated out, and the two "light components" Ψ_{+v} are left as degrees of freedom. In the effective theory based on v', slightly different degrees of freedom, namely $\Psi_{-v'}$ are integrated out. One might think that it should not be possible to recover $\Psi_{+v'}$ from Ψ_{+v} (Note that in the case of a heavy particle effective theory for a scalar field, these problems do not appear because in that case there are no degrees of freedom which are integrated out.).

At the tree level, however, everything is certainly correct. We have checked explicitly that the tree level matching Lagrangian

$$\mathcal{L}^{\text{tree}} = \overline{\Psi}_{+v}\left[ivD + i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp\right]\Psi_{+v} \tag{14}$$

(expansion in $1/m$ is implied) is invariant under the transformation law (13). The calculation is somewhat lengthy, but straightforward. It is given here in Appendix A.

4 Comparing with Luke and Manohar's transformation

4.1 The difference between the transformations

Luke and Manohar propose the following transformation law for the spinor Ψ_{+v} under reparameterization transformations [10]:

$$\Psi_{+v}(x) \to \Psi_{+v'}^{\text{LM}}(x) = e^{im\delta v\cdot x}\Lambda(v', \hat{u})\Lambda(v, \hat{u})^{-1}\Psi_{+v}(x) \tag{15}$$

where

$$\hat{u}^\mu = \frac{v^\mu + \dfrac{iD^\mu}{m}}{\sqrt{1 + \dfrac{2iv\cdot D}{m} - \dfrac{D^2}{m^2}}} \tag{16}$$

and

$$\Lambda(w, v) = \frac{1 + \not{w}\not{v}}{\sqrt{2(1 + v\cdot w)}} \tag{17}$$

Expanding this up to $O(1/m)$, we find

$$\begin{aligned}\Psi_{+v'}^{\text{LM}} &= \left[1 + im\delta v\cdot x + \frac{\delta\not{v}}{2} + \frac{i}{4m}\Big(\delta\not{v}(\not{D} - v\cdot D) - D\cdot\delta v\Big) + O\left(\frac{1}{m^2}\right)\right]\Psi_{+v} \\ &= \left[-\frac{i}{4m}D\cdot\delta v + O\left(\frac{1}{m^2}\right)\right]\Psi_{+v'}^{\text{Ch}} \end{aligned}\tag{18}$$

4.2 Velocity-operator notation

Such a change in the reparameterization transformation may be induced in a simple way, because besides transforming the fields, a reparameterization transformation *also* changes the four-velocity v^μ.

At this point, it is mnemonically useful to adopt a notation in which the incorporation of different v^μ into the Hilbert space of the theory is made explicit. This will make clear what happens when a transformation that changes v^μ acts in the middle of a string of operators that depend on v^μ.

Define Ψ_+ to be a column vector consisting of all of the heavy quark fields Ψ_{+v}. (The + reminds us that the field is a HQET field that satisfies $\not{v}\Psi_+ = \Psi_+$. All four-velocities are included in it, but not heavy antiquark fields, which would have to be dealt with separately, though analogously).

Then v^μ may be treated as a four-vector *operator* $\hat{v}^\mu$ that acts on Ψ_+. Its eigenspaces consist of states of definite four-velocity with eigenvalue v^μ. $\delta\hat{v}^\mu(\epsilon_i)$ is also an operator. It commutes with $\hat{v}^\mu$, and is defined in terms of the $\hat{v}^\mu$ operator via the formula for the change in four-velocity under an infinitesimal Lorentz transformation specified by the six infinitesimal parameters ϵ_i. (These could be boost rapidities and Euler angles, or any other convenient parameterization. What matters is that, unlike δv^μ, they do not depend on the value of v^μ). It is the boost parameters which actually specify the reparameterization transformation.

The shift in velocity is now accomplished explicitly by a shifting operator $\hat{S}(\epsilon_i) = \delta_{v',v+\delta v(\epsilon_i)}$, which obeys the commutation relations

$$\begin{aligned} [\hat{v}^\mu, \hat{S}(\epsilon_i)] &= \hat{S}(\epsilon_i)\delta\hat{v}^\mu(\epsilon_i) \\ \left[\delta\hat{v}^\mu(\epsilon_i), \hat{S}(\epsilon_i)\right] &= 0 \end{aligned} \tag{19}$$

for infinitesimal ϵ_i. Now everything about a reparameterization transformation, including the shift in four-velocity, is included in the action of the transformation operator on the field Ψ_+. We can handle both field and velocity transformations by manipulating operators in the usual way.

4.3 A field redefinition

Rewritten in velocity-operator notation (with the hats on the various operators omitted), Chen's reparameterization transformation to order $\frac{1}{m}$ is

$$\begin{aligned} M^{\mathrm{Ch}}(\epsilon_i)\Psi_+ &= S(\epsilon_i)\left[1 + im\delta v(\epsilon_i)\cdot x + \frac{\delta\not{v}(\epsilon_i)}{2} \right. \\ &\qquad \left. + \frac{\delta\not{v}(\epsilon_i)}{4m} i(\not{D} - v\cdot D) + O\left(\frac{1}{m^2}\right)\right]\Psi_+ \end{aligned} \tag{20}$$

and Luke and Manohar's is

$$M^{\mathrm{LM}}(\epsilon_i)\Psi_+ = S(\epsilon_i)\left[1 + im\delta v(\epsilon_i)\cdot x + \frac{\delta\not{v}(\epsilon_i)}{2} + \frac{\delta\not{v}(\epsilon_i)}{4m}i(\not{D} - v\cdot D) - \frac{i}{4m}D\cdot\delta v(\epsilon_i) + O\left(\frac{1}{m^2}\right)\right]\Psi_+ \quad (21)$$

where $S(\epsilon_i)$, v^μ, and $\delta v^\mu(\epsilon_i)$ are now understood to be operators.

Consider the field-redefinition operator

$$R\Psi_+ = \left[1 - \frac{i}{4m}D\cdot v\right]\Psi_+ \quad (22)$$

The important thing about R is that it is completely independent of ϵ_i, so it may be applied to Ψ_+ even in situations that have nothing to do with reparameterization transformations. It is a valid means of redefining fields so as to obtain one formulation of HQET from another.

Then applying R to Chen's transformation reveals that

$$\begin{aligned} RM^{\mathrm{Ch}}(\epsilon_i)\Psi_+ &= RM^{\mathrm{Ch}}(\epsilon_i)R^{-1}R\Psi_+ \\ &= M^{\mathrm{Ch}}(\epsilon_i)R\Psi_+ + S(\epsilon_i)\left[-\frac{i}{4m}D\cdot\delta v(\epsilon_i)\right]R\Psi_+ \\ &= M^{\mathrm{LM}}(\epsilon_i)R\Psi_+ + O\left(\frac{1}{m^2}\right) \end{aligned} \quad (23)$$

Even though the difference between Chen's transformation and Luke and Manohar's appears to depend on $\delta v(\epsilon_i)$, the shift-independent field redefinition R turns Chen's transformation into Luke and Manohar's, to order $1/m$. The redefined field $R\Psi$ transforms under Luke and Manohar's reparameterization transformation.

The field redefinition R is not a symmetry of the Lagrangian. However, since it is proportional to $D\cdot v$, the changes that it induces in the Lagrangian are manifestly class II operators. In fact, it is precisely the field redefinition necessary to absorb the class II operator $-\frac{1}{2m}(D\cdot v)^2$ in the Lagrangian obtained from tree-level matching, when the Lagrangian is written in terms of the field $R\Psi_+$.

The field redefinition necessary to absorb order $1/m$ and order $1/m^2$ class II operators in the Lagrangian obtained from tree-level matching is

$$R'\Psi_+ = \left[1 - \frac{iD\cdot v}{4m} + \frac{3(iD\cdot v)^2}{32m^2}\right]\Psi_+ \quad (24)$$

In general, a derivative term at order $1/m^j$ might affect the form of the reparameterization transformation at order $1/m^{j-1}$, because of the order m term in the reparameterization transformation. In this case, however, this does not happen, and the extra term in R' has no effect on the reparameterization transformation to order $1/m$.

(In [14] the field redefinition shown is the inverse of (24), because of notational conventions. Here we define the new Lagrangian to be the original expression written in terms of the transformed fields.)

Luke and Manohar's transformation, at least when expanded to first order in $1/m$, is a symmetry, not of the tree-level matching Lagrangian, but of the tree-level Lagrangian with the class II operators removed. Chen's transformation, on the other hand, is a symmetry of the tree-level Lagrangian with class II operators included. In Appendix B, it is demonstrated that Chen's transformation is not unique in this regard. There are other reparameterization transformations that preserve the entire tree-level matching Lagrangian to all orders in $1/m$.

5 Reparameterization invariance constraints on the effective Lagrangian

Reparameterization invariance leads to important constraints for the coupling constants in the effective Lagrangian. Due to the possiblity of field redefinitions, neither the form of the Lagrangian nor the form of the reparameterization transformation $\Psi_{+v} \to \Psi_{+v'}$ is unique. However, a field redefinition such as R above will induce only class II terms in the Lagrangian, and cannot change the constraints on the coefficients of class I terms in the Lagrangian.

5.1 Chen's transformation

Requiring invariance of the effective Lagrangian in (5) under Chen's tranformation law leads to the following constraints on the coefficients of the class I operators [14, 15, 16]

$$\begin{aligned} C_{kin} &= 1 \\ 2C_{mag} &= C_2 + 1 \end{aligned} \tag{25}$$

In addition, it sets the following constraint on the coefficients of some of the class II operators:

$$C_a = C_{D\cdot v} + \frac{1}{2} \tag{26}$$

Note that in [16], Chen, Kuang, and Oakes use an inconsisent basis that is non-Hermitian and does not include all of the necessary class II operators. They derive a spurious reparameterization constraint equating a class I coefficient with a class II coefficient ($c_4 = c_6$ in their paper). That this constraint is not gauge invariant was noted in [15].

5.2 Luke and Manohar's transformation

Luke and Manohar derived the same constraint for C_{kin}. When discussing the relationship between O_{mag} and O_2, they noted that the combination

$$O_{mag} + 2O_2 + O\left(\frac{1}{m^3}\right) \tag{27}$$

is reparameterization invariant, and that O_{mag} is not related to the leading-order Lagrangian by reparameterization invariance.

The second of these statements needs qualification. C_{mag} may be varied independently of the leading-order Lagrangian. However, the presence of the leading-order Lagrangian does modify the relationship between C_{mag} and C_2, because the reparameterization transformation acting on the leading-order Lagrangian yields a term at $O(1/m)$

$$\delta\mathcal{L}_0 = -\frac{g}{4m}\overline{\Psi}_{+v}\delta v_\mu \sigma^{\mu\nu} G_{\nu\rho} v^\rho \Psi_{+v} \tag{28}$$

which may only be cancelled by including a *difference* between C_{mag} and $2C_2$. This is why the constraint resulting from either Chen's transformation or Luke and Manohar's is actually $2C_{mag} = C_2 + 1$. The reparameterization invariance of (27) gives us the freedom to change C_{mag} and C_2 subject to this constraint without violating reparameterization invariance. It is easy to jump from the statements in [10] to the incorrect conclusion that $C_2 = 2C_{mag}$, but a close reading of [10] reveals that Luke and Manohar never actually state this, and it is not actually implied by what they do state. (Indeed, in [14], two of us jumped to exactly that erroneous conclusion, and then incorrectly reasoned that results from tree-level matching and one-loop running did not agree with the class I constraints from Luke and Manohar's transformation).

Luke and Manohar's transformation also induces the class II constraint

$$C_a = C_{D\cdot v} \tag{29}$$

This does not agree with the result of the usual tree-level matching procedure. However, it may be made to hold by a field redefinition, such as the one which sets $C_a = C_{D\cdot v} = 0$.

5.3 Field redefinitions

A general field redefinition $\Psi'_v = R\Psi_{+v}$ which preserves the projection property $P_v\Psi_{+v} = \Psi_{+v}$ and which transforms the class I part of the general effective Lagrangian into itself has the form

$$R\Psi_{+v} = \left[1 + \frac{a}{2m} iv\cdot D + \frac{b}{4m^2}D^2 + \frac{c}{4m^2}\sigma_{\mu\nu}D^\mu D^\nu + \frac{d}{4m^2}(iv\cdot D)^2 + O(\frac{1}{m^3})\right]\Psi_{+v} \tag{30}$$

where a, b, c, d are complex numbers. The field redefinitions (22) and (24) are redefinitions of this type. It is straightforward to check that this transformation applied to the general effective Lagrangian in (5) does not change the class I terms. Applying such redefinitions to a reparameterization symmetry $M(\epsilon_i)$ yields a family of reparameterization transformations $RM(\epsilon_i)R^{-1}$ which preserve the class I constraints.

This does not generalize to higher orders; at $1/m^3$, the coefficients of the class I terms may change under field redefinitions unless the form of the field redefinitions is restricted further (however, the transformation (22) induces only class II terms to all orders).

6 Loops, matching, and running

The effective theory does not have the same short distance behavior as full QCD. This must be taken into account by introducing suitable matching corrections. In this section, we show that Chen's RPI symmetry still holds when this matching is performed to order $\frac{1}{m^2}$, but to all orders in α_s.

6.1 Comparing with explicit running calculations

It has been checked explicitly that renormalization of the effective Lagrangian at one loop does fulfill the class I constraints in (25) [14, 15]. Other class I running calculations ([17], and the revised version of [13]) give equivalent results for running of the class I operators at order $1/m^2$. [18] does not report results for the running for C_1, but does give a result for C_2 which agrees with the above and with (25).

In fact, the results for class II operators calculated in [15] and [17]) also obey the class II constraint from Chen's transformation, (26). Translated into the operator basis of [15], (26) becomes

$$\frac{1}{2}C_1^{(2)} + C_3^{(2)} - C_3^{(1)} = \frac{1}{2} \tag{31}$$

where we have used $C_3^{(2)} = C_4^{(2)}$ by Hermiticity. This identity is satisfied at tree level ($C_1^{(2)} = -1, C_3^{(2)} = 0, C_3^{(1)} = -1$ in their operator basis). It is also maintained by their calculated one-loop running, independently of their background field gauge-fixing parameter. Agreement with (26) is more manifest in [17], since their class I operator basis is more similar to the one we are using.

This raises the question of whether these constraints are preserved more generally under quantum corrections, beyond tree-level matching and one-loop running.

6.2 Spinors and 1PI Green's functions

The general prescription for matching one theory to another at some momentum scale is to ensure that the 1PI Green's functions of the two theories describe the same physics at that scale, in an expansion in inverse powers of the effective theory cutoff. The same transitions must possess the same amplitudes when expanded in this way.

Spinors that appear on external legs of Feynman diagrams are always solutions in momentum space of the unperturbed equation of motion. The free field equations for the quark fields are different in QCD and HQET, since parts of the quark-quark Green's function that arise from the leading equation of motion in full QCD are attributed to higher-order "interaction" terms in HQET.

Therefore, the spinors one puts on external legs in QCD are not the same as the ones used in HQET for the same physical situation. To find the Dirac spinor in terms of the corresponding HQET spinor, one substitutes $p^\mu = mv^\mu + k^\mu$ into the solutions of the momentum-space free-field Dirac equation, and writes the resulting expression in terms of a HQET spinor u_{+v} for which $\not{v} u_{+v} = u_{+v}$. For a HQET spinor u_{+v}, the corresponding QCD spinor is

$$u_{QCD} = \left[1 + \frac{1}{2m + k \cdot v} \left(\not{k} - k \cdot v\right)\right] u_{+v} \tag{32}$$

The calculation may be simplified by putting external quark momenta on shell. This is necessary so that, later, we can use form-factor decompositions to say things about operator coefficients. It also transforms factors such as $k \cdot v$ into higher-order quantities in $\frac{1}{m^2}$, simplifying the series expansion to finite order. Taking $(mv + k)^2 = m^2$ for external quarks gives the modified matching relation

$$u_{QCD} = \left[1 + \frac{1}{2m - k^2/(2m)} \left(\not{k} + \frac{k^2}{2m}\right)\right] u_{+v} \tag{33}$$

and allows factors elsewhere in the 1PI Green's functions to be similarly moved to higher orders in $1/m$.

This procedure has the disadvantage of slightly complicating the calculation of coefficients of "class II operators" which vanish according to the free-field equation of motion in the effective theory. It does not make it impossible to say anything about such operators, since we are making the spinors obey the full theory's free-field equation of motion, rather than making the fields obey the effective theory's coupled equation of motion. Some remnants of these operators will remain, but putting the quark momenta on shell will give some Feynman vertices of different class II operators the same form, so that we can only say things about linear combinations of them.

6.3 Gauge invariance

When matching at tree level, it was possible to maintain gauge invariance explicitly at all steps of the calculation. This is because, at tree level, the generating functional of 1PI Green's functions is identical to the Lagrangian. Therefore, one can match Lagrangians, deal with fields instead of spinors, and use covariant derivatives instead of momenta.

When calculating loop diagrams, on the other hand, it is necessary to choose a gauge. Gauge invariance can be made somewhat explicit by using background field gauge, but diagrams will still treat interactions with different numbers of gluons as separate vertices, and in the spinor-matching procedure above we treat momenta separately from gluon couplings. The consequences of gauge invariance then reappear later in the form of Ward identities relating different Green's functions to one another. We will make use of one such identity when proving that Chen's RPI symmetry holds under loop matching corrections to order $\frac{1}{m^2}$.

6.4 Regularization scheme

Since a matching prescription does not involve the infrared divergent terms in a theory, the regularization scheme used for infrared divergences does not matter, as long as it is used consistently in the two theories. Thus we can use dimensional regularization to regularize both ultraviolet and infrared divergences [4, 19]. When used with $\overline{\mathrm{MS}}$, this eliminates all loop diagrams that do not possess a mass scale other than the renormalization scale μ. This includes all loop diagrams in HQET, since there the quark mass becomes a factor in coupling constants rather than a contribution to the propagator.

Therefore, using this regularization scheme eliminates the need to calculate HQET loop diagrams when matching to any order in perturbation theory. We calculate 1PI loop diagrams to any desired order in full QCD, with external quarks on shell and all divergences dimensionally regularized; apply the spinor substitution

(33); and adjust the couplings in the HQET Lagrangian so that the derived 1PI Green's function arises *at tree level*.

Using this regularization scheme affords us an opportunity to prove relations to all orders in α_s. Lorentz and parity invariance of full QCD allow us to write its 1PI Green's functions, with all loop corrections included, in terms of invariant form factors. If the Green's functions in HQET may be computed at tree level, then the structure of the full QCD Green's functions directly implies constraints upon the coupling constants of the HQET Lagrangian. To order $\frac{1}{m^2}$, it is sufficient to consider the 1PI quark-quark and quark-quark-gluon Green's functions in QCD.

6.5 The quark two-point function

The matching of the quark two-point function just corresponds to what we already know about tree-level matching of free fields. Loops can only yield mass and field renormalizations in the full theory, so after these divergences have been subtracted off with counterterms, the amputated 1PI Green's function is

$$i\bar{u}_{QCD}(\not{q} - m)u_{QCD} \tag{34}$$

where q^μ is the full momentum of the quark. Making the substitution (33), and simplifying the result using the projection identity $\not{v}u_{+v} = u_{+v}$, yields the two-point function for HQET:

$$\begin{aligned}\Gamma_{\bar{q}q} \;\; = \;\; & i\bar{u}_{+v}\left[1 + \frac{1}{2m - k^2/(2m)}\left(\not{k} + \frac{k^2}{2m}\right)\right](m\not{v} + \not{k} - m) \\ & \left[1 + \frac{1}{2m - k^2/(2m)}\left(\not{k} + \frac{k^2}{2m}\right)\right]u_{+v}\end{aligned} \tag{35}$$

This determines the coupling of every operator in HQET which contains a two-quark Feynman vertex with no gluons. To order $1/m^2$, applying the usual projection identities for heavy quark spinors, it is simply

$$i\bar{u}_{+v}\left(k\cdot v + \frac{k^2}{2m}\right)u_{+v} + O\left(\frac{1}{m^3}\right) \tag{36}$$

and it ends up enforcing the RPI constraint $C_{kin} = 1$. It will also constrain the coefficients of many high-order operators such as $\dfrac{1}{m^{2n-1}}\overline{\Psi}_{+v}(D^2)^n\Psi_{+v}$.

6.6 The quark-quark-gluon three-point function

The quark-quark-gluon vertex function is more interesting, because there can be quantum corrections to the structure in p^2, where p^μ is the transferred momentum.

However, considerations of Lorentz invariance and parity limit the 1PI vertex function in a manner familiar from QED. There is a Dirac form factor F_1 and a Pauli form factor F_2, which can depend on the momenta only via p^2:

$$\Gamma^{\mu a}_{\bar{q}gq} = ig\,\bar{u}'\gamma^\mu T^a u\,F_1(p^2, g, m, \mu) - \frac{g}{2m}\,\bar{u}'\sigma^{\mu\nu}T^a u\,p_\nu\,F_2(p^2, g, m, \mu) \tag{37}$$

Furthermore, $F_1(p^2 = 0) = 1$, because of gauge invariance. $F_2(p^2 = 0)$, giving the "anomalous chromomagnetic moment," is not constrained by symmetry and can be affected by loop corrections.

Regularizing all loop divergences with dimensional regularization, making the substitution (33), and applying $\not{v}u_{+v} = u_{+v}$ as above gives the *tree level* vertex function in HQET. To order $1/m^2$, where p^μ is the transferred momentum and k'^μ is the final residual momentum of the heavy quark, it is

$$\begin{aligned}
&ig\,\bar{u}'_{+v}T^a u_{+v}\,v^\mu\,F_1 + \frac{ig}{2m}\,\bar{u}'_{+v}T^a u_{+v}\,(2k'^\mu - p^\mu)\,F_1 \\
&-\frac{g}{2m}\,\bar{u}'_{+v}\sigma^{\mu\alpha}T^a u_{+v}\,p_\alpha\,(F_1 + F_2) \\
&+\frac{ig}{8m^2}\,\bar{u}'_{+v}T^a u_{+v}\,v^\mu p^2\,(F_1 + 2F_2) + \frac{ig}{8m^2}\,\bar{u}'_{+v}T^a u_{+v}\,v^\mu[k'^2 + (k'-p)^2]\,F_1 \\
&+\frac{g}{4m^2}\,\bar{u}'_{+v}\sigma^{\alpha\beta}T^a u_{+v}\,k'_\alpha p_\beta v^\mu\,(F_1 + 2F_2)
\end{aligned} \tag{38}$$

The term that goes like $k^2 + (k-p)^2$ at order $1/m^2$ is a contribution from class II operators. It looks like the Feynman vertices of O_a, but because external momenta are on shell, it can also arise from $O_{D\cdot v}$. The remaining terms all come from the class I operators.

Expanding the form factors as $F_i(p^2) = F_{i0} + p^2/m^2 F_{i2} + O(1/m^4)$ makes it possible to read off the coefficients directly, with some ambiguity in the case of the class II operators:

$$\begin{aligned}
C_{kin} &= F_{10} \\
C_{mag} &= F_{10} + F_{20} \\
C_1 &= F_{10} + 8F_{12} + 2F_{20} \\
C_2 &= F_{10} + 2F_{20} \\
C_a - C_{D\cdot v} &= \frac{1}{2}F_{10}
\end{aligned} \tag{39}$$

(Comparing with the Feynman rules listed in the long e-print version of [14], it is evident that terms in the Feynman vertices of O_1 and O_2 with factors of $p \cdot v$ have vanished here. This is again because of the on-shell quark momenta, which promote these terms to order $1/m^3$.)

This procedure yields no constraints on C_1, and, to this order, Chen's RPI does not constrain it either. Applying the Ward identity $F_{10} = 1$ yields Chen's RPI constraints

$$\begin{aligned} C_{kin} &= 1 \\ 2C_{mag} &= C_2 + 1 \\ C_a &= C_{D\cdot v} + \frac{1}{2} \end{aligned} \tag{40}$$

Of course, the first relation already followed from the two-point function. That it shows up here as well is a consequence of gauge symmetry.

6.7 Running under the renormalization group

In heavy quark effective field theory, we typically want to know the values of coefficients at some momentum scale which is far below the scale where matching to the full theory is done. After matching to the full theory to some order in the number of loops, one uses the renormalization group equation to determine how the coefficients in the effective field theory Lagrangian evolve under large changes in scale. The anomalous dimensions to use are typically calculated using diagrams with one more loop than was used in matching.

As described in Section 6.1, it is known that running at one loop preserves the reparameterization invariance constraints to order $1/m^2$. The result derived above implies that the class I constraints should apply for arbitrary numbers of loops. This is because renormalization group running can be seen as a special case of the matching procedure, which includes the terms from arbitrarily large orders in loops which dominate when the scale is far below the matching scale. If the RPI constraints apply at all orders in loops upon matching, they must therefore also apply to the coefficients found by running under the renormalization group. Therefore, we have shown not only that class I reparameterization invariance constraints apply to order $1/m^2$ upon matching to full QCD, but that they apply under renormalization group running as well, to all orders in α_s.

Which transformation is actually a symmetry of the Lagrangian depends on the class II terms, and therefore on how the quark fields are defined. It is useful, as in [14], to eliminate class II terms at all stages of matching and running. One starts with the Lagrangian with class II terms absorbed by a field redefinition. Then the renormalization group running incorporates a field redefinition that continuously absorbs class II terms induced by the running. Under these conditions (if the class I operators are defined according to our operator definitions), Luke and Manohar's transformation is a symmetry of the Lagrangian to order $1/m^2$, since it is a symmetry of a Lagrangian that satisfies the class I constraints and has no class II terms.

On the other hand, if the heavy quark fields are defined by tree-level matching in the usual way, and are not redefined to remove class II operators (as in most existing renormalization calculations, such as [13, 15, 17, 18]), then our results imply that Chen's constraints on the class II coefficients also hold under running, to order $1/m^2$ and to all orders in α_s. Then Chen's transformation is a symmetry of the renormalized Lagrangian to order $1/m^2$.

7 Conclusions

The form of a reparameterization transformation may be modified by conjugating it with other symmetry transformations, or with field redefinitions that affect the coefficients of class II operators. We have demonstrated that the forms of reparameterization invariance advocated by Chen and by Luke and Manohar are both members of a the resulting family of viable reparameterization transformations. Of the two, only Chen's is a member of the more restricted family of symmetries of the entire Lagrangian derived from tree-level matching.

Both transformations induce the *same* constraints on class I operator coefficients to order $1/m^2$. We have proven that these constraints hold not only at tree level, but to all orders in α_s, upon matching between HQET and QCD and under renormalization-group running. The transformations, in this sense, are symmetries of the quantum theory as well as the classical theory. The constraint imposed by Chen's transformation on the class II operators also holds to order $1/m^2$ and all orders in α_s, if the fields are not redefined to remove or modify the class II terms.

8 Acknowledgements

We would like to thank Markus A. Luty for very important discussions, and for his notes on reparameterization invariance and baryons in the $1/N$ expansion (unpublished). We would also like to thank Thorsten Ohl and Christopher Balzereit for suggesting that we consider the case of the class II constraints. Further thanks are due to Pilar Hernandez, Aneesh V. Manohar, and Raman Sundrum.

M.F. would like to thank the members of the Theoretical Physics Group at Harvard University for their kind hospitality. This work is supported in part by the National Science Foundation (Grant #PHY-9218167) and by the Deutsche Forschungsgemeinschaft.

A Invariance of $\mathcal{L}^{\rm tree}$

The tree level matching Lagrangian is given by

$$\mathcal{L}^{\rm tree} = \overline{\Psi}_{+v} A(v) \Psi_{+v} \tag{41}$$

where

$$A(v) = ivD + i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp \tag{42}$$

We want to prove invariance under the transformation

$$\begin{aligned}
& v \to v + \delta v \\
& \Psi_{+v} \to \left[1 + im\delta vx + \frac{\delta\not{v}}{2} + \frac{\delta\not{v}}{2}\frac{1}{2m+ivD} i\not{D}_\perp\right]\Psi_{+v} \\
& \overline{\Psi}_{+v} \to \overline{\Psi}_{+v}\left[1 - im\delta vx + \frac{\delta\not{v}}{2} + i\not{D}_\perp \frac{1}{2m+ivD}\frac{\delta\not{v}}{2}\right]
\end{aligned} \tag{43}$$

Now

$$\delta\mathcal{L} = \delta\overline{\Psi}_{+v} A \Psi_{+v} + \overline{\Psi}_{+v} A \delta\Psi_{+v} + \overline{\Psi}_{+v} \delta A \Psi_{+v} \tag{44}$$

Here

$$\begin{aligned}
P_{+v}\delta A P_{+v} = {} & P_{+v}\delta v^\mu \Big[iD_\mu - iD_\mu \frac{1}{2m+ivD} i\not{D}_\perp - i\not{D}_\perp \frac{1}{2m+ivD} iD_\mu \frac{1}{2m+ivD} i\not{D}_\perp \\
& - i\not{D}_\perp \frac{1}{2m+ivD} iD_\mu\Big] P_{+v} \\
= {} & P_{+v}\left\{i\delta vD - i\not{D}_\perp \frac{1}{2m+ivD} i\delta vD \frac{1}{2m+ivD} i\not{D}_\perp\right\} P_{+v}
\end{aligned} \tag{45}$$

where we have used

$$\frac{\partial}{\partial v^\mu}\frac{1}{2m+ivD} = \frac{1}{2m+ivD}(-iD^\mu)\frac{1}{2m+ivD} \tag{46}$$

Now consider

$$\delta\overline{\Psi}_{+v} A \Psi_{+v} + \overline{\Psi}_{+v} A \delta\Psi_{+v} = \overline{\Psi}_{+v}\{[A, im\delta vx] + A\delta M + \overline{\delta M} A\}\Psi_{+v} \tag{47}$$

where

$$\delta M = \frac{\delta\not{v}}{2} + \frac{\delta\not{v}}{2}\frac{1}{2m+ivD} i\not{D}_\perp \tag{48}$$

Now

$$[A, im\delta vx] = [ivD + i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp, im\delta vx] \tag{49}$$

Firstly

$$[ivD, im\delta vx] = [iv\partial, im\delta vx] - g[vA, im\delta vx] = 0 \tag{50}$$

because of $v\delta v = 0$. Secondly we have

$$[i\not{D}_\perp, im\delta vx] = [i\not{D}, im\delta vx]$$

$$= [i\not{\partial}, im\delta vx] = -m\delta\not{v} \tag{51}$$

Thirdly

$$[\frac{1}{2m + ivD}, im\delta vx] = 0 \tag{52}$$

And so we obtain

$$[A, im\delta vx] = [i\not{D}_\perp \frac{1}{2m + imD} i\not{D}_\perp, im\delta vx]$$

$$= -mi\not{D}_\perp \frac{1}{2m + ivD} \delta\not{v} - m\delta\not{v} \frac{1}{2m + imvD} i\not{D}_\perp \tag{53}$$

Furthermore

$$A\delta M = ivD\frac{\delta\not{v}}{2} + \frac{\delta\not{v}}{2}\frac{ivD}{2m + ivD} i\not{D}_\perp + i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp \frac{\delta\not{v}}{2}$$

$$+ i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp \frac{\delta\not{v}}{2} \frac{1}{2m + ivD} i\not{D}_\perp \tag{54}$$

Now

$$P_{+v} ivD \frac{\delta\not{v}}{2} P_{+v} = 0$$

$$P_{+v} i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp \frac{\delta\not{v}}{2} P_{+v} = P_{+v} i\not{D}_\perp \frac{1}{2m + ivD} iP_{-v}\not{D}_\perp P_{-v} \frac{\delta\not{v}}{2} P_{+v}$$

$$= P_{+v} i\not{D}_\perp \frac{-ivD}{2m + ivD} \delta\not{v} P_{+v} \tag{55}$$

and so

$$P_{+v} A\delta M P_{+v} = P_{+v}\Big[\frac{\delta\not{v}}{2}\frac{ivD}{2m + ivD} i\not{D}_\perp - i\not{D}_\perp \frac{ivD}{2m + ivD}\delta\not{v}$$

$$+ i\not{D}_\perp \frac{1}{2m + ivD} i\not{D}_\perp \frac{\delta\not{v}}{2}\frac{1}{2m + ivD} i\not{D}_\perp\Big] P_{+v} \tag{56}$$

Similarly

$$P_{+v}\overline{\delta M}AP_{+v} = P_{+v}\Big[-\delta\not{v}\frac{ivD}{2m+ivD}i\not{D}_\perp + i\not{D}_\perp\frac{ivD}{2m+ivD}\frac{\delta\not{v}}{2}$$
$$+ i\not{D}_\perp\frac{1}{2m+ivD}\frac{\delta\not{v}}{2}i\not{D}_\perp\frac{1}{2m+ivD}i\not{D}_\perp \tag{57}$$

and finally (sandwiching between a pair of P_{+v}'s is implied)

$$\begin{aligned} A\delta M + \overline{\delta M}A &= -\frac{\delta\not{v}}{2}\frac{ivD}{2m+ivD}i\not{D}_\perp - i\not{D}_\perp\frac{ivD}{2m+ivD}\frac{\delta\not{v}}{2} \\ &\quad + i\not{D}_\perp\frac{1}{2m+ivD}i\delta vD\frac{1}{2m+ivD}i\not{D}_\perp \end{aligned}\tag{58}$$

Plugging everything together, we find the variation of the Lagrangian:

$$\begin{aligned} \delta\mathcal{L} &= \overline{\Psi}_{+v}\Big\{-mi\not{D}_\perp\frac{1}{2m+ivD}\delta\not{v} - m\delta\not{v}\frac{1}{2m+ivD}i\not{D}_\perp - \frac{\delta\not{v}}{2}\frac{ivD}{2m+ivD}i\not{D}_\perp \\ &\quad - i\not{D}_\perp\frac{ivD}{2m+ivD}\frac{\delta\not{v}}{2} + i\not{D}_\perp\frac{1}{2m+ivD}i\delta vD\frac{1}{2m+ivD}i\not{D}_\perp + i\delta vD \\ &\quad - i\not{D}_\perp\frac{1}{2m+ivD}i\delta vD\frac{1}{2m+ivD}i\not{D}_\perp\Big\}\Psi_{+v} \\ &= \overline{\Psi}_{+v}\Big\{i\delta vD - \frac{\delta\not{v}}{2}\frac{ivD+2m}{2m+ivD}i\not{D}_\perp - i\not{D}_\perp\frac{ivD+2m}{2m+ivD}\frac{\delta\not{v}}{2}\Big\}\Psi_{+v} \\ &= \overline{\Psi}_{+v}\Big\{i\delta vD - i\delta vD\Big\}\Psi_{+v} = 0 \end{aligned}\tag{59}$$

which concludes the proof.

B Is Chen's transformation unique?

In this appendix we will show that Chen's transformation law is not unique even in the restricted family of symmetries of the full Lagrangian derived from tree-level matching.

So let's try to find the class of all reparameterization transformation which leave the Lagrangian

$$\mathcal{L}^{\rm tree}(v,\Psi_{+v}) = \overline{\Psi}_{+v}A(v)\Psi_{+v} \tag{60}$$

invariant ($A(v)$ has been given in the previous section). So consider an infinitesimal transformation

$$v \to v' = v + \delta v \tag{61}$$

with

$$\delta v \cdot v = 0 \tag{62}$$

What is the most general ansatz for the transformation $\Psi_{+v} \to \Psi_{+v'}$? The field $\Psi_{+v'}$ must have two properties: (i) it must have the correct projection property $P_{+v'}\Psi_{+v'} = \Psi_{+v'}$ and (ii) the derivative acting on it must produce the correct residual momentum k' instead of k. The most general ansatz compatible with these two requirements is

$$\Psi_{+v'} = \Big[1 + im\delta v \cdot x\Big] P_{+v'} B \Psi_{+v} \tag{63}$$

where B must not depend explicitly on x, but is otherwise arbitrary. B is a matrix in Dirac space and will contain covariant derivatives. Define

$$B_{\pm} := P_{\pm v} B \tag{64}$$

Then

$$\Psi_{+v'} = \left[\left(1 + im\delta v \cdot x + \frac{\delta \not{v}}{2}\right) B_+ + \frac{\delta \not{v}}{2} B_-\right] \Psi_{+v} \tag{65}$$

For $\delta v \to 0$, we must have $\Psi_{+v'} = \Psi_v$. Therefore we can assume B_- to be of lowest order in δv, i.e. $B_- = O(\delta v)^0$ and $B_+ = 1 + \delta B_+$, where $\delta B_+ = O(\delta v)$. And so

$$\Psi_{+v'} = \left[1 + \delta B_+ + im\delta v \cdot x + \frac{\delta \not{v}}{2} + \frac{\delta \not{v}}{2} B_-\right] \Psi_{+v} \tag{66}$$

At this point one can notice that the only combination of B_+ and B_- which enters the transformation law is

$$\delta B_+ + \frac{\delta \not{v}}{2} B_-$$

but not B_+ or B_- themselves.

Using the various tricks and techniques of the previous section, one can now inserte this general transformation into the Lagrangian, and require its variation to vanish. Defining

$$C_- := B_- - \frac{1}{2m + iv \cdot D} i \not{D}_\perp \tag{67}$$

this finally leads to

$$0 = \delta \mathcal{L} = \overline{\Psi}_{+v} \left\{ A(v) \left[\delta B_+ + \frac{\delta \not{v}}{2} C_-\right] + \left[\overline{\delta B_+} + \overline{C_-} \frac{\delta \not{v}}{2}\right] A(v) \right\} \Psi_{+v} \tag{68}$$

A solution to this equation is $\delta B_+ = 0$ and $C_- = 0$. This is Chen's transformation. Are there other solutions?

Firstly note again, that only $\delta B_+ + \frac{\delta v}{2} B_-$ enters in the transformation law, i.e. only the sum $\delta B_+ + \frac{\delta v}{2} C_-$ matters and solutions with $\delta B_+ + \frac{\delta v}{2} C_- = 0$ do not lead to different reparameterization transformations.

Secondly, however, there are solutions with $\delta B_+ + \frac{\delta v}{2} C_- \neq 0$. A non-trivial example is

$$\delta B_+ + \frac{\delta \not{v}}{2} C_- = i \frac{i\delta v \cdot D}{m} \frac{A(v)}{m} \tag{69}$$

Note that this transformation is 'class II' in a generalized sense, i.e. it vanishes for classical solutions of the *full* tree level effective Lagrangian with $A(v)\Psi_{+v} = 0$.

References

[1] S. Nussinov and W. Wentzel, Phys. Rev. **D39** (1987) 130;
M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45** (1987) 292; Sov. J. Nucl. Phys. **47** (1988) 511

[2] N. Isgur and M. B. Wise, Phys. Lett. **B208** (1988) 504; Phys. Lett. **B232** (1989) 113; Phys. Lett. **B237** (1990) 527;
H. D. Politzer and M. B. Wise, Phys. Lett. **B206** (1988) 681; **B208** (1988) 504

[3] W. E. Caswell and G. P. Lepage, Phys. Lett. **B167** (1986) 437;
G. P. Lepage and B. A. Thacker, Nucl. Phys. **B4** (Proc. Suppl.) (1988) 199

[4] E. Eichten and B. Hill, Phys. Lett. **B234** (1990) 511

[5] H. Georgi, Phys. Lett. **B240** (1990) 447

[6] B. Grinstein, Nucl. Phys. **B339** (1990) 253

[7] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343** (1990) 1

[8] H. Georgi and M. B. Wise, Phys. Lett. **B243** (1990) 279

[9] M. J. Dugan, M. Golden, and B. Grinstein, Phys. Lett. **B282** (1992) 142

[10] M. Luke and A. V. Manohar, Phys. Lett. **B286** (1992) 348

[11] Y.-Q. Chen, Phys. Lett. **B317** (1993) 421

[12] W. Kilian and T. Ohl, Phys. Rev. **D50** (1994) 4649

[13] C. L. Y. Lee, CALT-68-1663 (1991) (unpublished). Note that there exists a revised version of this preprint, in which a variety of errors have been corrected. The results in this revised version agree with those in [12, 14, 15, 17].

[14] M. Finkemeier and M. McIrvin, Phys. Rev. **D55** (1997) 377. A longer, more detailed version is available as hep-ph/9607272; HUTP-96/A026.

[15] C. Balzereit and T. Ohl, Phys. Lett. **B386** (1996) 335

[16] Y.-Q. Chen, Y.-P. Kuang, and R. J. Oakes, Phys. Rev. **D52** (1995) 264

[17] B. Blok, J.G. Körner, D. Pirjol, and J.C. Rojas, hep-ph/9607233

[18] S. Balk, J.G. Körner, and D. Pirjol, Nucl. Phys. **B428** (1994) 499

[19] M. Neubert, Phys. Reports **245** (1994) 259